



Robust gain-scheduled controller design for uncertain LPV systems: Affine quadratic stability approach

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Abstract

Our paper deals with robust gain-scheduled controller design for uncertain LPV systems which ensures closed-loop stability and guaranteed cost for all scheduled parameter changes. The novel procedure is based on LPV paradigm, Lyapunov theory of stability and guaranteed cost from LQ theory. The feasible design procedures are obtained in the form of BMI or LMI. The class of control structure includes centralized or decentralized fixed order output feedbacks like PI controller. Numerical examples illustrate the effectiveness of the proposed approach.

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1. Introduction

Robust control theory for linear systems is well established, but almost all real processes are more or less nonlinear. If the operating region is small we can use robust control approaches to design a linear robust controller where the nonlinearities are treated as model uncertainties. However for real processes where the nonlinearities are large the above mentioned controller synthesis is inapplicable. For this reason controller design for nonlinear systems is very determinative and important field of research in today's time.

Gain-scheduling is one of the most common used controller design approach for nonlinear systems and has a widely range of use in industrial application, in process control and in aerospace technology. In this paper a novel linear parameter-varying (LPV) based gain-scheduling controller design for real parameter uncertainty is proposed.

Consider a linear parameter varying system with state space matrices which are fixed functions of known vector parameter varying $\alpha(t)$. This model can be a linear time invariant (LTI) plant model which is result from linearization

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of the non-linear plants along trajectories of the known parameter $\alpha(t) \in \langle \underline{\alpha}, \bar{\alpha} \rangle$. In this note the following LPV system will be used

$$\begin{aligned}\dot{x}(t) &= A(\alpha(t))x(t) + B(\alpha(t))u(t) \\ y(t) &= Cx(t)\end{aligned}\tag{1}$$

where

$$A(\alpha(t)) = A_0 + \sum_{i=1}^p A_i \alpha_i(t), \quad B(\alpha(t)) = B_0 + \sum_{i=1}^p B_i \alpha_i(t)$$

and $x \in R^n$ is the state, $u \in R^m$ is a control input, $y = R^l$ is the measurement output vector, $A_0, B_0, A_i, B_i, i = 1, 2, \dots, p, C$ are constant matrices of appropriate dimension, $\alpha(t) \in \langle \underline{\alpha}, \bar{\alpha} \rangle \in \Omega$ vector of time-varying plant parameters.

Consider a real LPV system matrices with uncertainty in the form

$$\begin{aligned}A_{\Delta}(\alpha(t), \beta(t)) &= A(\alpha(t)) + \Delta_A(\beta(t)) \\ B_{\Delta}(\alpha(t), \beta(t)) &= B(\alpha(t)) + \Delta_B(\beta(t))\end{aligned}\tag{2}$$

where

$$\Delta_A(\beta(t)) = \sum_{j=1}^{N_u} A_{u j} \beta_j, \quad \Delta_B(\beta(t)) = \sum_{j=1}^{N_u} B_{u j} \beta_j$$

$$\beta_j \in \langle \underline{\beta_j}, \bar{\beta_j} \rangle$$

where N_u is the number of uncertainty and $A_{u j}, B_{u j}$ are constant matrices with appropriate dimension. Substituting (2) to (1) we obtain the following uncertain LPV system

$$\begin{aligned}\dot{x}(t) &= A_{\Delta}(\theta(t))x(t) + B_{\Delta}(\theta(t))u(t) \\ y(t) &= Cx(t)\end{aligned}\tag{3}$$

where

$$\begin{aligned}A_{\Delta}(\theta(t)) &= A_{\Delta 0} + \sum_{i=1}^{p+N_u} A_{\Delta i} \theta_i(t) \\ B_{\Delta}(\theta(t)) &= B_{\Delta 0} + \sum_{i=1}^{p+N_u} B_{\Delta i} \theta_i(t)\end{aligned}$$

$$\theta_i \in \langle \underline{\theta_i}, \bar{\theta_i} \rangle$$

Let we denote

$$\begin{array}{llll} A_{\Delta 0} & = & A_0, & B_{\Delta 0} & = & B_0, \\ A_{\Delta 1} & = & A_1, & B_{\Delta 1} & = & B_1, \\ & \vdots & & & \vdots & \\ A_{\Delta p} & = & A_p, & B_{\Delta p} & = & B_p, \\ A_{\Delta p+1} & = & A_{u 1}, & B_{\Delta p+1} & = & B_{u 1}, \\ & \vdots & & & \vdots & \\ A_{\Delta p+N_u} & = & A_{u N_u}, & B_{\Delta p+N_u} & = & B_{u N_u}, \end{array}$$

$$\begin{aligned}
\theta_1 &= \alpha_1 \\
\theta_2 &= \alpha_2 \\
&\vdots \\
\theta_p &= \alpha_p \\
\theta_{p+1} &= \beta_1 \\
&\vdots \\
\theta_{p+N_u} &= \beta_{N_u}
\end{aligned}$$

The main motivation for our work lies in [Sato \(2011\)](#), [Köroğlu \(2010\)](#), [Wang and Balakrishnan \(2002\)](#), [Leith and Leithhead \(2000\)](#), [Rugh and Shamma \(2000\)](#) and [Veselý and Ilka \(2013\)](#). In the paper [Sato \(2011\)](#) the author tackles the design problem of gain scheduled controllers for LPV systems via parameter-dependent Lyapunov function. Recently, [Köroğlu \(2010\)](#) proposed the design method for gain scheduled problem using a similar technique to [Sato \(2011\)](#). Improved stability analysis and gain scheduled controller synthesis for parameter-dependent systems are proposed in [Wang and Balakrishnan \(2002\)](#). Survey of scheduled controller analysis and synthesis are presented in papers [Leith and Leithhead \(2000\)](#) and [Rugh and Shamma \(2000\)](#). In the paper [Veselý and Ilka \(2013\)](#) the authors presented a novel gain-scheduled controller design approach which ensures closed-loop stability and guaranteed cost for all scheduled parameter changes.

In this note our approach is based on

- A consideration of the uncertain LPV systems (3), scheduling parameters θ_i , $i = 1, 2, \dots, p + N_u$ and their derivatives with respect to time are supposed to lie in a priori given hyper rectangles, $\theta \in \Omega$.
- Quadratic stability (QS) introduced by [Gahinet et al. \(1996\)](#), because quadratic stability allows arbitrarily fast parameter variations.
- We use the notion of guaranteed cost to guarantee the performance of closed-loop system ([Veselý and Ilka, 2013](#)).
- The class of control structure includes centralized or decentralized fixed order output feedback like PI controller.

The paper is organized as follows. Section 2 brings preliminaries and problem formulation. The main result is presented in Section 3. In Section 4, numerical example illustrate the effectiveness of the proposed approach.

2. Preliminaries and problem formulation

Consider an uncertain LPV system in the form (3). The output feedback control law is considered for PI controller in the form

$$u(t) = F(\theta(t))y = F(\theta(t))Cx \quad (4)$$

where

$$F(\theta(t)) = F_0 + \sum_{i=1}^p F_i \theta_i$$

is a static output feedback gain-scheduled matrix for PI controller. Note that number of controller matrices is only p , the rest N_u is equal to zero. Substituting (4) to (3) and after some manipulation we can obtain the closed-loop system in the following form

$$\dot{x} = A_c(\theta(t))x \quad (5)$$

where

$$A_c(\theta(t)) = A_\Delta(\theta(t)) + B_\Delta(\theta(t))F(\theta(t))C$$

To access the performance quality a quadratic cost function (Engwerda and Weeren, 2008) known from LQ theory is often used. In this note the original quadratic cost function is used, where weighting matrices depends on scheduling parameters (Ilka and Veselý, 2013). Using this approach we can affect on performance quality in each working point separately. The quadratic cost function is in the form

$$J(\theta(t)) = \int_0^\infty (x^T Q(\theta(t))x + u^T R u) dt \quad (6)$$

where

$$Q(\theta(t)) = Q_0 + \sum_{i=1}^p Q_i \theta_i, \quad Q_i = Q_i^T \geq 0$$

and $R > 0$. The guaranteed cost is defined in a standard way.

Definition 1. Consider the system (3) with control algorithm (4). If there exists a control law u^* and a positive scalar J^* such that the closed-loop system (5) is stable and the value of closed-loop cost function (6) satisfies $J \leq J^*$ then J^* is said to be a guaranteed cost and u^* is said to be guaranteed cost control law for system (3).

Definition 2. The linear closed-loop system (5) for $\theta \in \Omega$ is quadratically stable if and only if there exist a symmetric positive definite matrix $P > 0$ and for the first derivative of Lyapunov function $V(\theta(t)) = x^T P x$ along the trajectory of closed-loop system (5) holds (Apkarian et al., 1995)

$$\frac{dV(x, \theta)}{dt} = A_c(\theta)^T P + P A_c(\theta) < 0 \quad (7)$$

From LQ theory we introduce the well known results.

Lemma 1. Consider the closed-loop system (5). Closed-loop system (5) is quadratically stable with guaranteed cost if and only if the following inequality holds

$$B_e(\theta(t)) = \min_u \left\{ \frac{dV(\theta(t))}{dt} + J(\theta(t)) \right\} \leq 0 \quad (8)$$

for all $\theta \in \Omega$.

3. Main results

In this section the robust gain scheduled controller design procedure which guarantees the quadratic stability and guaranteed cost for $\theta \in \Omega$ is presented. The main results for the case of gain-scheduled closed-loop stability analysis reduce to LMI condition and for gain scheduled controller synthesis to BMI one, which can be linearized.

The main result of this section the robust gain scheduled design procedure relies in the concept of multi-convexity, that is, convexity along each direction θ_i of the parameter space. The implications of multicovexity for scalar quadratic functions are given in the next lemma (Gahinet et al., 1996).

Lemma 2. Consider a scalar quadratic function of $\theta \in R^p$.

$$f(\theta_1, \dots, \theta_p) = a_0 + \sum_{i=1}^p a_i \theta_i + \sum_{i=1}^p \sum_{j>i}^p b_{ij} \theta_i \theta_j + \sum_{i=1}^p c_i \theta_i^2 \quad (9)$$

and assume that $f(\theta_1, \dots, \theta_p)$ is multi-convex, that is

$$\frac{\partial^2 f(\theta)}{\partial \theta_i^2} = 2c_i \geq 0 \quad (10)$$

for $i = 1, 2, \dots, p$. Then $f(\theta)$ is negative for all $\theta \in \Omega$ and $\dot{\theta} \in \Omega_t$ if and only if it takes negative values at the corners of θ .

Using [Lemma 2](#) the following theorem is obtained

Theorem 1. *Closed-loop system (5) is quadratically stable with guaranteed cost if there exist positive defined $P > 0$ for all $\theta(t) \in \Omega$, matrices $Q_i, R, i = 1, 2, \dots, p$ and gain-scheduled controller matrices $F(\theta(t))$ satisfying*

$$M(\theta(t)) < 0; \quad \theta(t) \in \Omega \quad (11)$$

$$M_{ddii} \geq 0; \quad i = 1, 2, \dots, p + N_u \quad (12)$$

where

$$M(\theta(t)) = M_0 + \sum_{i=1}^p M_i \theta_i(t) + \sum_{i=1}^p \sum_{j=i}^p M_{ij} \theta_i(t) \theta_j(t) \quad (13)$$

$$M_{ddii} = C^T F_i^T B_{\Delta i}^T P + P B_{\Delta i} F_i C + C^T F_i^T R F_i C \quad (14)$$

furthermore

$$\begin{aligned} M_0 &= \begin{bmatrix} W_{110} & W_{210}^T \\ W_{210} & W_{220} \end{bmatrix} \\ M_i &= \begin{bmatrix} W_{11i} & W_{21i}^T \\ W_{21i} & W_{22i} \end{bmatrix} \\ M_{ij} &= \begin{bmatrix} W_{11ij} & W_{21ij}^T \\ W_{21ij} & W_{22ij} \end{bmatrix} \\ W_{110} &= A_{\Delta 0}^T P + P A_{\Delta 0} + Q_0 - P B_{\Delta 0} R^{-1} B_{\Delta 0}^T P \\ W_{11i} &= A_{\Delta i}^T P + P A_{\Delta i} + Q_i - P B_{\Delta i} R^{-1} B_{\Delta 0}^T P \\ &\quad - P B_{\Delta 0} R^{-1} B_{\Delta i}^T P \\ W_{11ij} &= -P B_{\Delta i} R^{-1} B_{\Delta j}^T P \\ W_{210} &= F_0 C + R^{-1} B_{\Delta 0} P \\ W_{21i} &= F_i C + R^{-1} B_{\Delta i} P \\ W_{21ij} &= 0 \\ W_{220} &= -R^{-1} \\ W_{22i} &= 0 \\ W_{22ij} &= 0 \end{aligned}$$

Proof. Proof is based on [Lemma 1](#) and [2](#). From (8) we can obtain

$$M(\theta(t)) = A_c(\theta(t))^T P + P A_c(\theta(t)) + Q(\theta(t)) + C^T F(\theta(t))^T R F(\theta(t)) C \leq 0 \quad (15)$$

If we substitute $G(\theta(t)) = F(\theta(t))C + R^{-1}B(\theta(t))P$ to (15), after some manipulation and using Schur complement, we obtain

$$M(\theta(t)) = \begin{bmatrix} M_{11}(\theta(t)) & M_{21}(\theta(t))^T \\ M_{21}(\theta(t)) & M_{22}(\theta(t)) \end{bmatrix} < 0 \quad (16)$$

where

$$M_{11}(\theta(t)) = A_{\Delta}(\theta(t))^T P + P A_{\Delta}(\theta(t)) + Q(\theta(t)) - P B_{\Delta}(\theta(t)) R^{-1} B_{\Delta}(\theta(t))^T P$$

$$M_{21}(\theta(t)) = G(\theta(t))$$

$$M_{22}(\theta(t)) = -R^{-1}$$

If we extend (16) to affine form, we can obtain (23). After we extend (15) to affine form we can obtain inequality (12), where M_{ddii} is the second derivation of (15) by θ_i

$$\frac{\partial^2 M(\theta(t))}{\partial \theta_i^2} = M_{ddii} \geq 0 \quad (17)$$

We can linearize the nonlinear part of (23) and (24) to obtain LMI controller design procedure.

$$\begin{aligned} \text{lin}(-PB_{\Delta}(\theta(t))R^{-1}B_{\Delta}(\theta(t))^T P) &\leq -XB_{\Delta}(\theta(t))R^{-1}B_{\Delta}(\theta(t))^T P - PB_{\Delta}(\theta(t))R^{-1}B_{\Delta}(\theta(t))^T X \\ &\quad + XB_{\Delta}(\theta(t))R^{-1}B_{\Delta}(\theta(t))^T X \end{aligned} \quad (18)$$

where in each iteration pores $X=P$. We can rewrite (24) using Schur complement to this form

$$\begin{bmatrix} L_{11} & L_{21}^T \\ L_{21} & L_{22} \end{bmatrix} \geq 0 \quad (19)$$

where

$$\begin{aligned} L_{11} &= PP + C^T F_i^T B_{\Delta i}^T B_{\Delta i} F_i C + C^T F_i^T R F_i C \\ L_{21} &= P - B_{\Delta i} F_i C \\ L_{22} &= I \end{aligned}$$

Linearization of the nonlinear parts of L_{11}

$$\text{lin}(PP) \leq PX + XP - XX \quad (20)$$

where in each iteration pores $X=P$. Note, that X is the same as in (18)

$$\text{lin}(C^T F_i^T B_{\Delta i}^T B_{\Delta i} F_i C) \leq C^T Z_i^T B_{\Delta i}^T B_{\Delta i} F_i C + C^T F_i^T B_{\Delta i}^T B_{\Delta i} Z_i C - C^T Z_i^T B_{\Delta i}^T B_{\Delta i} Z_i C \quad (21)$$

$$\text{lin}(C^T F_i^T R F_i C) \leq C^T Z_i^T R F_i C + C^T F_i^T R Z_i C - C^T Z_i^T R Z_i C \quad (22)$$

where in each iteration pores $Z_i = F_i$. Using this linearization the following theorem is obtained

Theorem 2. *Closed-loop system (5) is quadratically stable with guaranteed cost if there exist positive defined $P > 0$ for all $\theta(t) \in \Omega$, matrices $Q_i, R, i = 1, 2, \dots, p$ and gain-scheduled controller matrices $F(\theta(t))$ satisfying (11) and (12) where*

$$M(\theta(t)) = M_0 + \sum_{i=1}^p M_i \theta_i(t) + \sum_{i=1}^p \sum_{j=i}^p M_{ij} \theta_i(t) \theta_j(t) \quad (23)$$

$$M_{ddii} = \begin{bmatrix} L_{11} & L_{21}^T \\ L_{21} & L_{22} \end{bmatrix} \quad (24)$$

furthermore

$$\begin{aligned}
 M_0 &= \begin{bmatrix} W_{110} & W_{210}^T \\ W_{210} & W_{220} \end{bmatrix} \\
 M_i &= \begin{bmatrix} W_{11i} & W_{21i}^T \\ W_{21i} & W_{22i} \end{bmatrix} \\
 M_{ij} &= \begin{bmatrix} W_{11ij} & W_{21ij}^T \\ W_{21ij} & W_{22ij} \end{bmatrix} \\
 L_{11} &= PX + XP - XX + C^T Z_i^T B_{\Delta i}^T B_{\Delta i} F_i C \\
 &\quad + C^T F_i^T B_{\Delta i}^T B_{\Delta i} Z_i C - C^T Z_i^T B_{\Delta i}^T B_{\Delta i} Z_i C \\
 &\quad + C^T Z_i^T R F_i C + C^T F_i^T R Z_i C - C^T Z_i^T R Z_i C \\
 L_{21} &= P - B_{\Delta i} F_i C \\
 L_{22} &= I \\
 W_{110} &= A_{\Delta 0}^T P + P A_{\Delta 0} + Q_0 - X B_{\Delta 0} R^{-1} B_{\Delta 0}^T P \\
 &\quad - P B_{\Delta 0} R^{-1} B_{\Delta 0}^T X + X B_{\Delta 0} R^{-1} B_{\Delta 0}^T X \\
 W_{11i} &= A_{\Delta i}^T P + P A_{\Delta i} + Q_i - X B_{\Delta i} R^{-1} B_{\Delta 0}^T P \\
 &\quad - X B_{\Delta 0} R^{-1} B_{\Delta i}^T P - P B_{\Delta i} R^{-1} B_{\Delta 0}^T X \\
 &\quad - P B_{\Delta 0} R^{-1} B_{\Delta i}^T X + X B_{\Delta 0} R^{-1} B_{\Delta i}^T X \\
 &\quad + X B_{\Delta i} R^{-1} B_{\Delta 0}^T X \\
 W_{11ij} &= -P B_{\Delta i} R^{-1} B_{\Delta j}^T P \\
 W_{210} &= F_0 C + R^{-1} B_{\Delta 0} P \\
 W_{21i} &= F_i C + R^{-1} B_{\Delta i} P \quad W_{21ij} = 0 \\
 W_{220} &= -R^{-1} \quad W_{22i} = 0 \quad W_{22ij} = 0
 \end{aligned}$$

4. Examples

The first example is taken from paper [Stewart \(2012\)](#). Consider a simple non-linear plant with parameter varying coefficients

$$\begin{aligned}
 \dot{x}(t) &= a(\alpha)x(t) + b(\alpha)u(t) \\
 y(t) &= x(t)
 \end{aligned} \tag{25}$$

where $\alpha(t) \in R$ is an exogenous signal that changes the parameters of the plant as follows

$$a(\alpha) = -6 - \frac{2}{\pi} \arctan\left(\frac{\alpha}{20}\right) \tag{26}$$

$$b(\alpha) = \frac{1}{2} + \frac{5}{\pi} \arctan\left(\frac{\alpha}{20}\right) \tag{27}$$

Let the problem to be a design gain scheduled PID controller which will guarantee the closed-loop stability and guaranteed cost for $\alpha \in \langle 10, 100 \rangle$. We will demonstrate that with gain scheduled controller we will obtain for closed-loop system practically identical behavior. To be able to demonstrate this feature, let us divide the working area to 3 sections so that in each area where the plant parameter changes they are nearly linear ([Fig. 1](#)). In these areas calculated

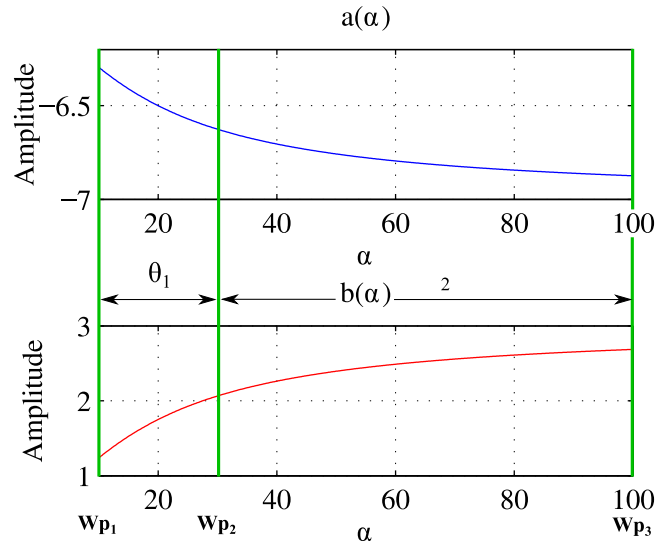


Fig. 1. Simulation results.

transfer functions we transform to time domain to obtain scheduling model in the form (3). The obtained model we extended for gain scheduled PI controller design. The extended model with added uncertainty is given as follows

$$\begin{aligned}
 A_0 &= \begin{bmatrix} -6.5848 & 0 \\ 1 & 0 \end{bmatrix}, & A_1 &= \begin{bmatrix} -0.1652 & 0 \\ 0 & 0 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} -0.1243 & 0 \\ 0 & 0 \end{bmatrix}, & A_{u1} &= \begin{bmatrix} 0.0500 & 0 \\ 0 & 0 \end{bmatrix}, \\
 A_{u2} &= \begin{bmatrix} -0.0900 & 0 \\ 0 & 0 \end{bmatrix}, & B_0 &= \begin{bmatrix} 1.9619 \\ 0 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} 0.4131 \\ 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.3108 \\ 0 \end{bmatrix}, \\
 B_{u1} &= \begin{bmatrix} -0.0600 \\ 0 \end{bmatrix}, & B_{u2} &= \begin{bmatrix} 0.0800 \\ 0 \end{bmatrix}, \\
 C &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & D &= 0
 \end{aligned}$$

Using Theorem 1 with weighting matrices $Q_i = q_i I$, $q_1 = 1 \times 10^{-10}$, $q_2 = q_3 = 1 \times 10^{-11}$, $R = rI$, $r = 1$, $\rho = 1 \times 10^2$, which is the upper constraint of Lyapunov matrix $P < \rho$, we obtain gain scheduled controller in the form:

$$G_{rGS} = G_{r0} + G_{r1}\theta_1 + G_{r2}\theta_2 \quad (28)$$

where

$$\begin{aligned}
 G_{r0} &= \frac{2.9423s + 21.1575}{s} \\
 G_{r1} &= -\frac{0.0606s + 0.4330}{s} \\
 G_{r2} &= \frac{1.1283s + 8.0673}{s}
 \end{aligned} \quad (29)$$

Simulation results (Fig. 2) confirm, that Theorem 1 holds, but we can see also that with used weighting matrices we do not obtain identical closed-loop behavior in each working point. To demonstrate the main feature of variable weighting let us to change the weighting matrices to get required performance quality. An another gain-scheduled controller in the

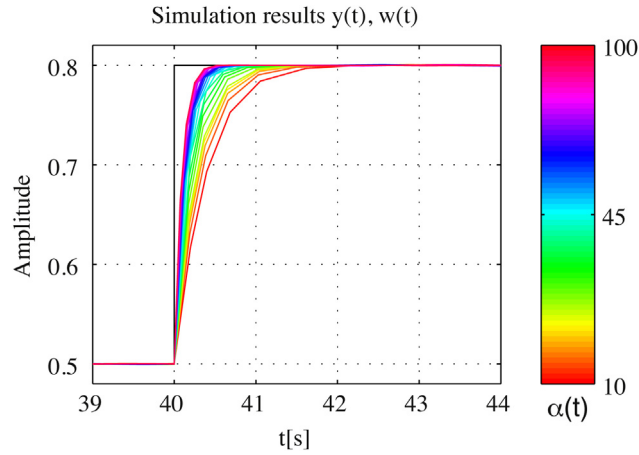


Fig. 2. Simulation results with GS controller (29).

form (28) was obtained using *Theorem 1* with weighting matrices $Q_i = q_i I$, $q_1 = 1 \times 10^{-10}$, $q_2 = q_3 = 1 \times 10^{-9}$, $R = rI$, $r = 1$, $\rho = 1 \times 10^2$. The obtained controller matrices are as follows

$$\begin{aligned} G_{r0} &= \frac{1.2576s + 12.7373}{s} \\ G_{r1} &= -\frac{0.3472s + 2.4912}{s} \\ G_{r2} &= -\frac{0.3345s + 2.4003}{s} \end{aligned} \quad (30)$$

Simulation results confirms that with variable weighting matrices we can affect performance quality separately in each working points and we can tune the system to the desired condition. In Fig. 3 the black line indicate the setpoint $w(t)$ and the colorized lines indicates the system outputs at different values of $\alpha(t)$.

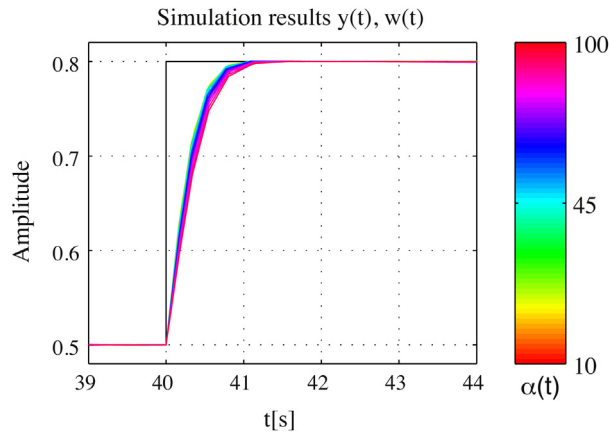


Fig. 3. Simulation results with GS controller (31).

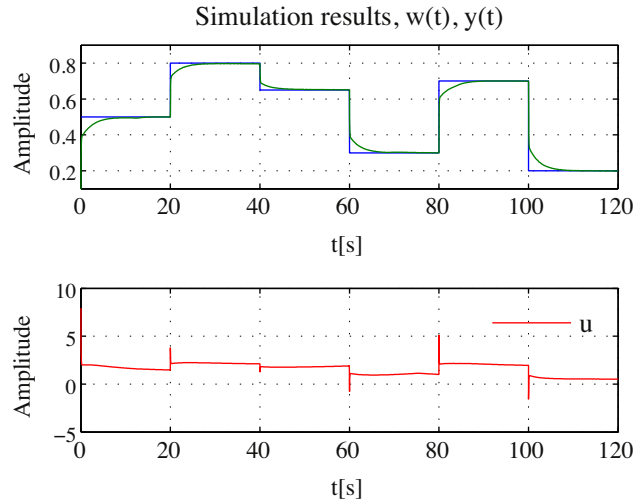


Fig. 4. Simulation results $w(t)$, $y(t)$, $\alpha \in (10, 100)$ linearly changing.

Using [Theorem 2](#) which is the LMI design procedure another gain-scheduled controller with weighting matrices $Q_i = q_i I$, $q_1 = 1 \times 10^{-10}$, $q_2 = q_3 = 1 \times 10^{-9}$, $R = rI$, $r = 1$, $\rho = 1 \times 10^2$ is obtained in the form (28) where gain-scheduled controllers are as follows

$$\begin{aligned} G_{r0} &= \frac{9.9123s + 5.0218}{s} \\ G_{r1} &= -\frac{3.7197s + 1.1768}{s} \\ G_{r2} &= -\frac{2.2514s + 0.8172}{s} \end{aligned} \quad (31)$$

Simulation results will illustrate the main benefit of quadratic stability which is that allows arbitrary fast model parameter changes. [Figs. 4 and 5](#) show the results if α is changing linearly from 10 to 100. [Figs. 6 and 7](#) show results when α is changing sinusously with frequency $f=0.1$ Hz. [Figs. 8 and 9](#) show results when α is changing sinusously with frequency $f=1$ Hz. [Figs. 10 and 11](#) show results when α is changing sinusously with frequency $f=10$ Hz. In [Figs. 4, 6, 8, 10](#) the green line indicate the system output $y(t)$, the blue line the setpoint $w(t)$ and the red line the controller output $u(t)$. In [Figs. 5, 7, 9 and 11](#) the green and blue lines indicates the calculated scheduling parameters $\theta_1(t)$ and $\theta_2(t)$ respectively and the magenta line indicate the exogenous signal $\alpha(t)$.

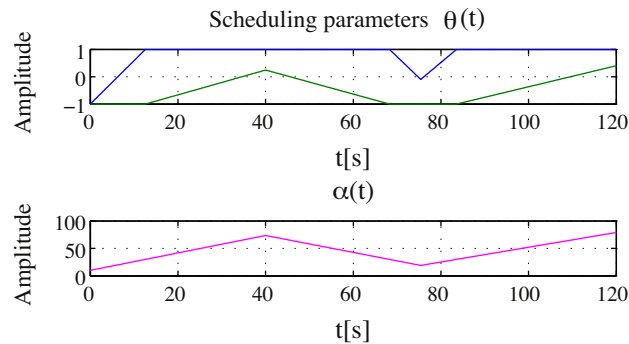


Fig. 5. Simulation results $\theta(t)$, $\alpha(t)$, $\alpha \in (10, 100)$ linearly changing.

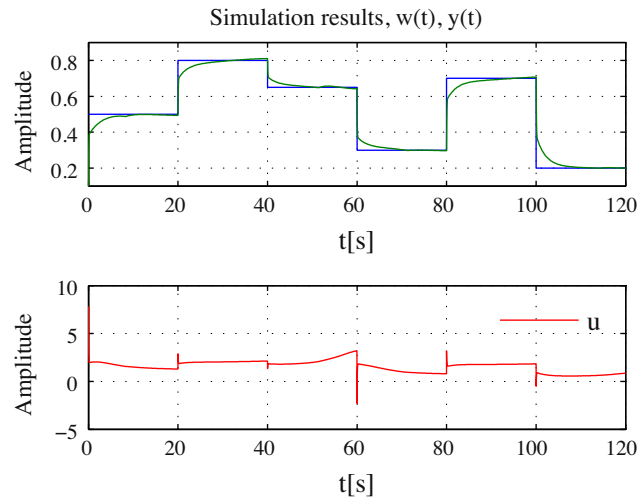


Fig. 6. Simulation results $w(t)$, $y(t)$, $\alpha \in \langle 10, 100 \rangle$ sinusously changing with $f=0.1$ Hz.

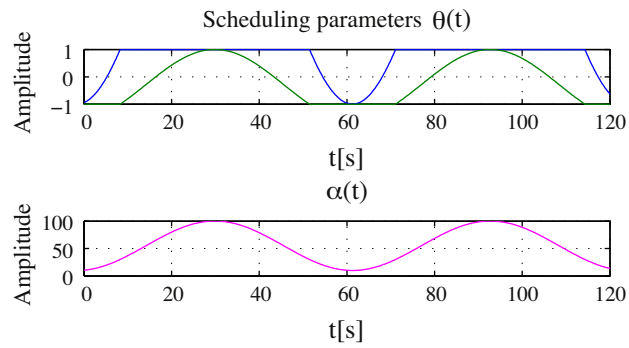


Fig. 7. Simulation results $\theta(t)$, $\alpha(t)$, $\alpha \in \langle 10, 100 \rangle$ sinusously changing with $f=0.1$ Hz.

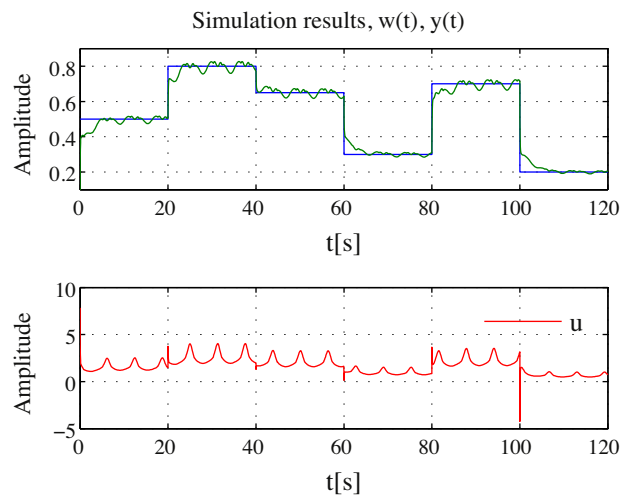


Fig. 8. Simulation results $w(t)$, $y(t)$, $\alpha \in \langle 10, 100 \rangle$ sinusously changing with $f=1$ Hz.

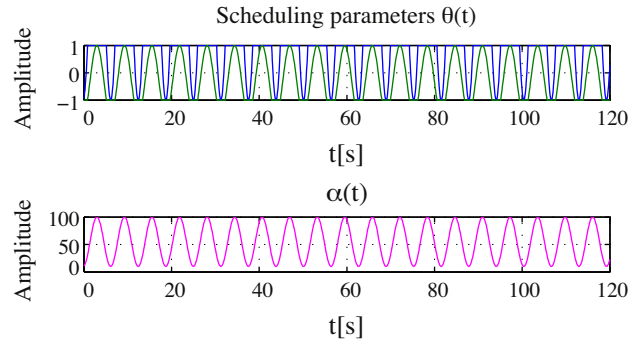


Fig. 9. Simulation results $\theta(t)$, $\alpha(t)$, $\alpha \in \langle 10, 100 \rangle$ sinusously changing with $f=1$ Hz.

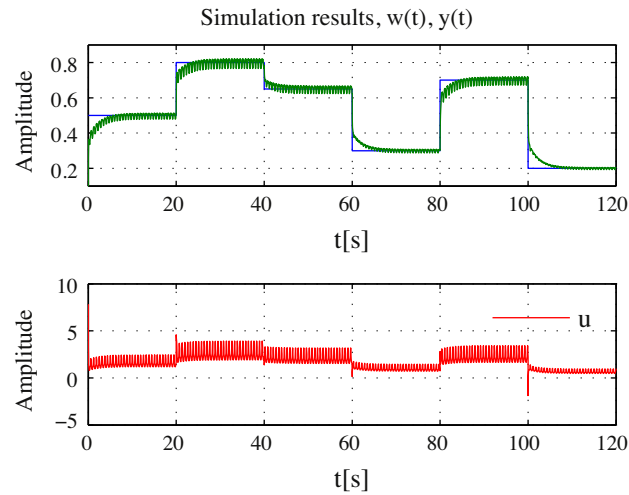


Fig. 10. Simulation results $w(t)$, $y(t)$, $\alpha \in \langle 10, 100 \rangle$ sinusously changing with $f=10$ Hz.

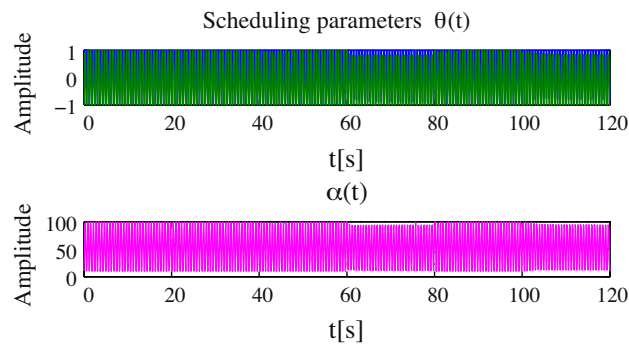


Fig. 11. Simulation results $\theta(t)$, $\alpha(t)$, $\alpha \in \langle 10, 100 \rangle$ sinusously changing with $f=10$ Hz.

5. Conclusion

The paper addresses the problem of the robust gain-scheduled controller design for uncertain LPV system which ensures the closed-loop stability and guaranteed cost for all scheduled parameter changes. The proposed original procedures are based on Lyapunov theory of stability, the notion of guaranteed cost, the LPV paradigm and BMI/LMI. Using original variable weighting matrices we can affect the performance quality separately in each working point and we can tune the system to the desired conditions through all parameter changes. The obtained results, illustrated

on examples show the applicability of the novel robust gain-scheduled design procedures. The obtained results are in the form of BMI and LMI and the class of control structure includes centralized or decentralized fixed order output feedback like PI controller. The obtained simulation results show that the robust gain-scheduled controller may give better performance than classical one including classical robust controller.

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